

ON BIPOLAR SOFT SETS

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ABSTRACT. We have studied the concept of bipolarity of information in the soft sets. We have defined bipolar soft sets and basic operations of union, intersection and complementation for bipolar soft sets. Examples of bipolar soft sets and an application of bipolar soft sets in a decision making problem with general algorithms have also been presented at the end.

1. INTRODUCTION

The idea of soft set and its applications was introduced by Molodtsov ^[1] in 1999. The concept was motivated by the limitations on parameterization process faced by the other theories while handling uncertain and fuzzy data. Molodtsov presented a new structure namely soft set accommodating the parameterization problem. He also provided a general idea to define operations on soft sets. Maji et al. ^[2] defined the operations of union and intersection on soft sets but Ali et al. ^[3] pointed out some basic problems occurring with the results using the definitions of [2] and defined some new operations on soft sets which were based upon the selection of parameters in particular. Ali et al. ^[4] studied the algebraic structures of soft sets under these new operations. The hybrid structures like fuzzy soft sets^[5], rough soft sets ^[6], ^[7], intuitionistic fuzzy soft sets^[8], vague soft sets ^[9], and interval valued fuzzy soft sets ^[10] were introduced and studied well through their applications in various fields.

We have mentioned earlier that the motivation for the idea of soft sets is the need of a parameterization tool and therefore, the role of parameters becomes important and considerable. If U is an initial universe and E is a set of parameter then a soft set defines a parameterized family of subsets of U . Molodtsov presented an example in [1] which discusses the approximation of "attractiveness" of some houses through different parameters like being expensive, beautiful, in the green surroundings etc. Each approximation was taken as a set of houses possessing the property as defined by the corresponding parameter. In that example the set of parameters was {expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair}. We observe that the parameters "expensive" and "cheap" and similarly "in good repair" and "in bad repair" are clearly interrelated. The presence or absence of one affects that of the other and so, a further filtration may be done to choose a reduced set of parameters. An intelligent knowledge-based system must be designed in a way that eliminates all the redundancies of input data and hence it should provide a maximized and efficient level of

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performance. Each parameter is a word or sentence and we need an efficient choice of parameters as well. How to choose a precise, reduced and sufficient set of parameters in order to optimize for a best possible decision? Chen et al. [11] discussed a parameterization reduction of soft sets by a step-wise reduction algorithm for parameters and compared it with the concept of attribute reduction in rough sets. More recently M. I. Ali^[12] has given a new method for the reduction of parameters in soft sets by keeping the equivalence of soft sets and information systems in view. However, these discussions are mainly about the roles of decision values i.e. the presence or absence of a particular parameter in a house and did not consider the definition of parameter's set itself. Our question has been somewhat answered by Dubois and Prade in [13] while considering the human judgments upon given pieces of information. They defined the role of polarity by saying that choices of people are usually caused by checking the good sides and the bad sides of alternatives separately. Then they choose according to whether the good or the bad sides are stronger. In other words the judgments also possess an intrinsic positive or negative flavor, what we call a polarity. Bipolarity refers to an explicit handling of positive and negative sides of information. Three types of bipolarity were discussed in [13] but we are using a rather generalized bipolarity here, dealing with the positive and negative impacts only.

First of all we have to differentiate between a word or sentence represented by a parameter and its negative (opposite) parameter e.g. "in good repair" and its opposite as "in bad repair". In [2] Maji et al. considered "not set" of parameters as {not expensive, not beautiful, not wooden, not cheap, not in the green surroundings, not modern, not in good repair, not in bad repair}^[2]. It is worth noting that the parameters "not cheap", "not in a good repair" and "expensive", "in a bad repair" are similar in meaning respectively! This indicates the problem while defining the set of parameters. In order to handle this problem we define a new concept of bipolar soft set. A bipolar soft set is obtained by considering not only a carefully chosen set of parameters but also an allied set of oppositely meaning parameters named as "Not set of parameters". Structure of a bipolar soft set is managed by two functions, say $F : A \rightarrow \mathcal{P}(U)$ and $G : \neg A \rightarrow \mathcal{P}(U)$ where $\neg A$ stands for the "not set of A " and G describes somewhat an opposite or negative approximation for the attractiveness of a house relative to the approximation computed by F . Maji et al.^[2] had used the "not set" to define the compliment of a soft set. The *complement of a soft set* simply gives the complements of the approximations. The above mentioned soft function G is rather more generalized than soft complement function and $(G, \neg A)^{[2]}$ can be any soft subset of $(F, A)^c$ ^[2]. The difference is the gray area of choice, that is, we may find some houses which do not satisfy any criteria properly e.g. A house may not be highly expensive but it does not assure its cheapness either. Thus, we must be careful while making our considerations for the parameterization of data keeping in view that, during approximations, there might be some indifferent elements in U .

We have given some preliminaries on soft sets and after that we define bipolar soft sets. We have defined operations of union and intersection for bipolar soft sets by taking restricted and extended sets of parameters. The set of parameters has been taken to be restricted by taking intersection and extended with the help of union and product operations. Examples are presented which elaborate the concepts and working methods for computations. The last section gives an application of bipolar

soft sets in decision making problems. We observe that the "Preference" is mostly viewed as an important concept in decision making and the choice parameters are not all equally attractive. Hence, it is crucial for a decision maker to examine these parameters in terms of their desirability. Weights may be added to parameters after comparing them on a common scale and thus we obtain a weighted table for a bipolar soft set. A revised form of decision making algorithm has also been presented and a comparison in results has been made to clarify the use of weights.

2. SOFT SETS

Let U be an initial universe, and E be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of U and A, B be non-empty subsets of E .

Definition 1. [1] A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow \mathcal{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) . Clearly, a soft set is not a set.

Definition 2. [3] For two soft sets (F, A) and (G, B) over a common universe U , we say that (F, A) is a soft subset of (G, B) if

- (1) $A \subseteq B$ and
- (2) $F(e) \subseteq G(e)$ for all $e \in A$.

We write $(F, A) \widetilde{\subset} (G, B)$.

(F, A) is said to be a soft super set of (G, B) , if (G, B) is a soft subset of (F, A) . We denote it by $(F, A) \widetilde{\supset} (G, B)$.

Definition 3. [2] Two soft sets (F, A) and (G, B) over a common universe U , are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

Definition 4. [2] Let $E = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, \dots, \neg e_n\}$ where, $\neg e_i = \text{not } e_i$ for all i .

Proposition 1. [2] For any subsets $A, B \subset E$,

- (1) $\neg(\neg A) = A$;
- (2) $\neg(A \cup B) = \neg A \cap \neg B$;
- (3) $\neg(A \cap B) = \neg A \cup \neg B$.

Definition 5. [2] The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where, $F^c : \neg A \rightarrow \mathcal{P}(U)$ is a mapping given by $F^c(e) = U - F(\neg e)$, for all $e \in \neg A$.

We call F^c to be the soft complement function of F . Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 6. [3] Union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C) , where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cup_{\mathcal{E}} (G, B) = (H, C)$.

Definition 7. [3] *The extended intersection of two soft sets (F, A) and (G, B) over a common universe U , is the soft set (H, C) where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases}.$$

We write $(F, A) \cap_{\mathcal{E}} (G, B) = (H, C)$.

Definition 8. [3] *Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_{\mathcal{R}} (G, B)$ and is defined as $(F, A) \cap_{\mathcal{R}} (G, B) = (H, A \cap B)$ where $H(e) = F(e) \cap G(e)$ for all $e \in A \cap B$.*

Definition 9. [2] *Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The restricted union of (F, A) and (G, B) is denoted by $(F, A) \cup_{\mathcal{R}} (G, B)$ and is defined as $(F, A) \cup_{\mathcal{R}} (G, B) = (H, C)$ where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cup G(e)$.*

Definition 10. [3] *Let U be an initial universe set, E be the set of parameters, and $A \subset E$.*

- (1) *(F, A) is called a relative null soft set (with respect to the parameter set A), denoted by Φ_A , if $F(e) = \emptyset$ for all $e \in A$.*
- (2) *(G, A) is called a relative whole soft set (with respect to the parameter set A), denoted by \mathfrak{U}_A , if $F(e) = U$ for all $e \in A$.*

3. BIPOLAR SOFT SETS

Let U be an initial universe and E be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of U and A, B, C are non-empty subsets of E . For the convenience of reading, we shall refer the reader to [1], [2] and [3] for basic concepts of soft sets and their properties. Now, we define

Definition 11. *A triplet (F, G, A) is called a bipolar soft set over U , where F and G are mappings, given by $F : A \rightarrow \mathcal{P}(U)$ and $G : \neg A \rightarrow \mathcal{P}(U)$ such that $F(e) \cap G(\neg e) = \emptyset$ (Empty Set) for all $e \in A$.*

In other words, a bipolar soft set over U gives two parametrized families of subsets of the universe U and the condition $F(e) \cap G(\neg e) = \emptyset$ for all $e \in A$, is imposed as a consistency constraint. For each $e \in A$, $F(e)$ and $G(\neg e)$ are regarded as the set of e -approximate elements of the bipolar soft set (F, G, A) . It is also observed that the relationship between the complement function and the defining function of a soft set is a particular case for the defining functions of a bipolar soft set, i.e. (F, F^c, A) is a bipolar soft set over U . The difference occurs due to the presence of uncertainty or hesitation or the lack of knowledge in defining the membership function. We name this uncertainty or gray area as the approximation for the degree of hesitation. Thus the union of three approximations i.e. e -approximation, $\neg e$ -approximation, and approximation of hesitation is U . We note that $\emptyset \subseteq U - \{F(e) \cup G(\neg e)\} \subseteq U$, for each $e \in A$. So, we may approximate the degree of hesitation in (F, G, A) by an allied soft set (H, A) defined over U , where $H(e) = U - \{F(e) \cup G(\neg e)\}$ for all $e \in A$.

Definition 12. *For two bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U , we say that (F, G, A) is a bipolar soft subset of (F_1, G_1, B) , if,*

- (1) $A \subseteq B$ and
- (2) $F(e) \subseteq F_1(e)$ and $G_1(\neg e) \subseteq G(\neg e)$ for all $e \in A$.

This relationship is denoted by $(F, G, A) \tilde{\subseteq} (F_1, G_1, B)$. Similarly (F, G, A) is said to be a *bipolar soft superset* of (F_1, G_1, B) , if (F_1, G_1, B) is a *bipolar soft subset* of (F, G, A) . We denote it by $(F, G, A) \tilde{\supseteq} (F_1, G_1, B)$.

Definition 13. Two bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U are said to be equal if (F, G, A) is a bipolar soft subset of (F_1, G_1, B) and (F_1, G_1, B) is a bipolar soft subset of (F, G, A) .

Definition 14. The complement of a bipolar soft set (F, G, A) is denoted by $(F, G, A)^c$ and is defined by $(F, G, A)^c = (F^c, G^c, A)$ where F^c and G^c are mappings given by $F^c(e) = G(\neg e)$ and $G^c(\neg e) = F(e)$ for all $e \in A$.

Definition 15. A bipolar soft set over U is said to be a relative null bipolar soft set, denoted by (Φ, \mathfrak{U}, A) if for all $e \in A$, $\Phi(e) = \emptyset$ and $\mathfrak{U}(\neg e) = U$, for all $e \in A$.

Definition 16. A bipolar soft set over U is said to be a relative absolute bipolar soft set, denoted by (\mathfrak{U}, Φ, A) , if for all $e \in A$, $\mathfrak{U}(e) = U$ and $\Phi(\neg e) = \emptyset$, for all $e \in A$.

Definition 17. If (F, G, A) and (F_1, G_1, B) are two bipolar soft sets over U then " (F, G, A) and (F_1, G_1, B) " denoted by $(F, G, A) \wedge (F_1, G_1, B)$ is defined by

$$(F, G, A) \wedge (F_1, G_1, B) = (H, I, A \times B)$$

where $H(a, b) = F(a) \cap F_1(b)$ and $I(\neg a, \neg b) = G(\neg a) \cup G_1(\neg b)$, for all $(a, b) \in A \times B$.

Definition 18. If (F, G, A) and (F_1, G_1, B) are two bipolar soft sets over U then " (F, G, A) or (F_1, G_1, B) " denoted by $(F, G, A) \vee (F_1, G_1, B)$ is defined by

$$(F, G, A) \vee (F_1, G_1, B) = (H, I, A \times B)$$

where $H(a, b) = F(a) \cup F_1(b)$ and $I(\neg a, \neg b) = G(\neg a) \cap G_1(\neg b)$, for all $(a, b) \in A \times B$.

Proposition 2. If (F, G, A) and (F_1, G_1, B) are two bipolar soft sets over U then

- (1) $((F, G, A) \vee (F_1, G_1, B))^c = (F, G, A)^c \wedge (F_1, G_1, B)^c$
- (2) $((F, G, A) \wedge (F_1, G_1, B))^c = (F, G, A)^c \vee (F_1, G_1, B)^c$.

Proof. Straightforward. □

Definition 19. Extended Union of two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the common universe U is the bipolar soft set (H, I, C) over U , where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ F_1(e) & \text{if } e \in B - A \\ F(e) \cup F_1(e) & \text{if } e \in A \cap B \end{cases}$$

$$I(\neg e) = \begin{cases} G(\neg e) & \text{if } e \in (\neg A) - (\neg B) \\ G_1(\neg e) & \text{if } e \in (\neg B) - (\neg A) \\ G(\neg e) \cap G_1(\neg e) & \text{if } e \in (\neg A) \cap (\neg B) \end{cases}$$

We denote it by $(F, G, A) \tilde{\cup} (F_1, G_1, B) = (H, I, C)$.

Definition 20. *Extended Intersection of two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the common universe U is the bipolar soft set (H, I, C) over U , where $C = A \cup B$ and for all $e \in C$,*

$$\begin{aligned} H(e) &= \begin{cases} F(e) & \text{if } e \in A - B \\ F_1(e) & \text{if } e \in B - A \\ F(e) \cap F_1(e) & \text{if } e \in A \cap B \end{cases} \\ I(\neg e) &= \begin{cases} G(\neg e) & \text{if } e \in (\neg A) - (\neg B) \\ G_1(\neg e) & \text{if } e \in (\neg B) - (\neg A) \\ G(\neg e) \cup G_1(\neg e) & \text{if } e \in (\neg A) \cap (\neg B) \end{cases} \end{aligned}$$

We denote it by $(F, G, A) \tilde{\cap} (F_1, G_1, B) = (H, I, C)$.

Definition 21. *Restricted Union of two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the common universe U is the bipolar soft set (H, I, C) , where $C = A \cap B$ is non-empty and for all $e \in C$*

$$H(e) = F(e) \cup G(e) \text{ and } I(\neg e) = F_1(\neg e) \cap G_1(\neg e).$$

We denote it by $(F, G, A) \cup_{\mathcal{R}} (F_1, G_1, B) = (H, I, C)$.

Definition 22. *Restricted Intersection of two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the common universe U is the bipolar soft set (H, I, C) , where $C = A \cap B$ is non-empty and for all $e \in C$:*

$$H(e) = F(e) \cap G(e) \text{ and } I(\neg e) = F_1(\neg e) \cup G_1(\neg e).$$

We denote it by $(F, G, A) \cap_{\mathcal{R}} (F_1, G_1, B) = (H, I, C)$.

Proposition 3. *Let (F, G, A) and (F_1, G_1, A) be two bipolar soft sets over a common universe U . Then the following are true*

- (1) $((F, G, A) \tilde{\cup} (F_1, G_1, B))^c = (F, G, A)^c \tilde{\cap} (F_1, G_1, B)^c$,
- (2) $((F, G, A) \tilde{\cap} (F_1, G_1, B))^c = (F, G, A)^c \tilde{\cup} (F_1, G_1, B)^c$,
- (3) $((F, G, A) \cup_{\mathcal{R}} (F_1, G_1, B))^c = (F, G, A)^c \cap_{\mathcal{R}} (F_1, G_1, B)^c$,
- (4) $((F, G, A) \cap_{\mathcal{R}} (F_1, G_1, B))^c = (F, G, A)^c \cup_{\mathcal{R}} (F_1, G_1, B)^c$.

Proof. 1) Let $e \in A \cup B$. There are three cases:

(i) If $e \in A - B$, then

$$\begin{aligned} (F \tilde{\cup} F_1)^c(e) &= (F(e))^c = (F^c \tilde{\cap} F_1^c)(e) \\ (G \tilde{\cup} G_1)^c(e) &= (G(e))^c = (G^c \tilde{\cap} G_1^c)(e), \end{aligned}$$

(ii) If $e \in B - A$, then

$$\begin{aligned} (F \tilde{\cup} F_1)^c(e) &= (F_1(e))^c = (F^c \tilde{\cap} F_1^c)(e) \\ (G \tilde{\cup} G_1)^c(e) &= (G_1(e))^c = (G^c \tilde{\cap} G_1^c)(e), \end{aligned}$$

(iii) If $e \in A \cap B$, then

$$\begin{aligned} (F \tilde{\cup} F_1)^c(e) &= (F(e) \cup F_1(e))^c = (F(e))^c \cap (F_1(e))^c \\ (G \tilde{\cup} G_1)^c(e) &= (G(e) \cap G_1(e))^c = (G(e))^c \cup (G_1(e))^c, \end{aligned}$$

and,

$$\begin{aligned} (F^c \tilde{\cap} F_1^c)(e) &= (F(e))^c \cap (F_1(e))^c \\ (G^c \tilde{\cap} G_1^c)(e) &= (G(e))^c \cup (G_1(e))^c, \end{aligned}$$

Therefore, in all three cases we obtain equality and thus

$$((F, G, A) \tilde{\cup} (F_1, G_1, B))^c = (F, G, A)^c \tilde{\cap} (F_1, G_1, B)^c.$$

The remaining parts can also be proved in a similar way. \square

Proposition 4. *If (Φ, \mathfrak{U}, A) is a null bipolar soft set, (\mathfrak{U}, Φ, A) an absolute bipolar soft set, and (F, G, A) , (F_1, G_1, A) are bipolar soft sets over U , then*

- (1) $(F, G, A) \tilde{\cup} (F_1, G_1, A) = (F, G, A) \cup_{\mathcal{R}} (F_1, G_1, A)$,
- (2) $(F, G, A) \tilde{\cap} (F_1, G_1, A) = (F, G, A) \cap_{\mathcal{R}} (F_1, G_1, A)$,
- (3) $(F, G, A) \tilde{\cup} (F, G, A) = (F, G, A)$; $(F, G, A) \tilde{\cap} (F, G, A) = (F, G, A)$,
- (4) $(F, G, A) \tilde{\cup} (\Phi, \mathfrak{U}, A) = (F, G, A)$; $(F, G, A) \tilde{\cap} (\Phi, \mathfrak{U}, A) = (\Phi, \mathfrak{U}, A)$,
- (5) $(F, G, A) \tilde{\cup} (\mathfrak{U}, \Phi, A) = (\mathfrak{U}, \Phi, A)$; $(F, G, A) \tilde{\cap} (\mathfrak{U}, \Phi, A) = (F, G, A)$.

Proof. Straightforward. \square

Example 1. *Let U be the set of houses under consideration, and E be the set of parameters, $U = \{h_1, h_2, h_3, h_4, h_5\}$,*

$E = \{e_1, e_2, e_3, e_4, e_5, e_6\} = \{ \text{in the green surroundings, wooden, cheap, in good repair, furnished, traditional} \}$. Then $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4, \neg e_5, \neg e_6\} = \{ \text{in the commercial area, marbled, expensive, in bad repair, non-furnished, modern} \}$. Suppose that $A = \{e_1, e_2, e_3, e_6\}$, and $B = \{e_2, e_3, e_4, e_5\}$. The bipolar soft sets (F, G, A) and (F_1, G_1, B) describe the “requirements of the houses” which Mr. X and Mr. Y are going to buy respectively. Suppose that

$$\begin{aligned} F(e_1) &= \{h_2, h_3\}, F(e_2) = \{h_1, h_2, h_5\}, F(e_3) = \{h_1, h_3\}, F(e_6) = \{h_2, h_3, h_5\} \\ G(\neg e_1) &= \{h_4, h_5\}, G(\neg e_2) = \{h_3, h_4\}, G(\neg e_3) = \{h_2, h_4\}, G(\neg e_6) = \{h_4\} \end{aligned}$$

and

$$\begin{aligned} F_1(e_2) &= \{h_2, h_5\}, F_1(e_3) = \{h_1, h_3, h_5\}, F_1(e_4) = \{h_1, h_3, h_4\}, F_1(e_5) = \{h_2, h_3\}, \\ G_1(\neg e_2) &= \{h_4\}, G_1(\neg e_3) = \{h_2, h_4\}, G_1(\neg e_4) = \{h_2\}, G_1(\neg e_5) = \{h_1, h_4\}. \end{aligned}$$

One may read this information as: Mr. X demands a house according to the parameters, “in the green surroundings, wooden, cheap and traditional”. The houses in the set U are parametrized accordingly, either having the said attribute or its opposite. We observe that Mr. X thinks that the houses h_2 and h_3 are situated in the green surroundings while h_4 and h_5 are situated in an entirely commercial area and we are not parameterizing h_1 by either of A or $\neg A$. It means that h_1 has some green surrounding area and some area which is either commercial or not green as such. This also shows that the approximations for $(G, \neg A)$ are not same as that of $(F, A)^c$ as defined in [2]. We note that, for Mr. Y , the houses h_2 and h_5 are wooden while h_4 is marbled whereas h_1 and h_3 are not sufficiently structured to be parametrized as either of these. We also see that Mr. X thinks that h_1 and h_3 are cheap while h_2 and h_4 are expensive but Mr. Y considers that h_1 , h_3 and h_4 are cheap while h_2 and h_5 are expensive ones. So parameterization differs according to any given situation and circumstances and further approximations about the given data may be obtained by applying the operations of union and intersection defined for bipolar soft sets.

Now, we approximate the resulting bipolar soft sets obtained by applying the above mentioned operations on (F, G, A) and (F_1, G_1, B) .

Let $(F, G, A) \tilde{\cup} (F_1, G_1, B) = (H_1, I_1, A \cup B)$. Then

$$\begin{aligned} H_1(e_1) &= \{h_2, h_3\}, H_1(e_2) = \{h_1, h_2, h_5\}, H_1(e_3) = \{h_1, h_3, h_5\}, \\ H_1(e_4) &= \{h_1, h_3, h_4\}, H_1(e_5) = \{h_2, h_3\}, H_1(e_6) = \{h_2, h_3, h_5\}, \end{aligned}$$

and

$$\begin{aligned} I_1(\neg e_1) &= \{h_4, h_5\}, I_1(\neg e_2) = \{h_4\}, I_1(\neg e_3) = \{h_2, h_4\}, I_1(\neg e_4) = \{h_4\}, \\ I_1(\neg e_5) &= \{h_1, h_4\}, I_1(\neg e_6) = \{h_4\}. \end{aligned}$$

Let $(F, G, A) \tilde{\cap} (F_1, G_1, B) = (H_2, I_2, A \cup B)$. Then

$$\begin{aligned} H_2(e_1) &= \{h_2, h_3\}, H_2(e_2) = \{h_2, h_5\}, H_2(e_3) = \{h_1, h_3\}, H_2(e_4) = \{h_1, h_3, h_4\}, \\ H_2(e_5) &= \{h_2, h_3\}, H_2(e_6) = \{h_2, h_3, h_5\}, \end{aligned}$$

and

$$\begin{aligned} I_2(\neg e_1) &= \{h_4, h_5\}, I_2(\neg e_2) = \{h_3, h_4\}, I_2(\neg e_3) = \{h_2, h_4\}, I_2(\neg e_4) = \{h_2\}, \\ I_2(\neg e_5) &= \{h_1, h_4\}, I_2(\neg e_6) = \{h_4\}. \end{aligned}$$

Let $(F, G, A) \cup_{\mathcal{R}} (F_1, G_1, B) = (H_3, I_3, A \cap B)$. Then

$$\begin{aligned} H_3(e_2) &= \{h_1, h_2, h_5\}, H_3(e_3) = \{h_1, h_3, h_5\} \quad \text{and} \\ I_3(\neg e_2) &= \{h_4\}, I_3(\neg e_3) = \{h_2, h_4\}. \end{aligned}$$

Let $(F, G, A) \cap_{\mathcal{R}} (F_1, G_1, B) = (H_4, I_4, A \cap B)$. Then

$$\begin{aligned} H_4(e_2) &= \{h_2, h_5\}, H_4(e_3) = \{h_1, h_3\} \quad \text{and} \\ I_4(\neg e_2) &= \{h_3, h_4\}, I_4(\neg e_3) = \{h_2, h_4\}. \end{aligned}$$

Let $(F, G, A) \vee (F_1, G_1, B) = (H_5, I_5, A \times B)$. Then

$$\begin{aligned} H_5(e_1, e_2) &= \{h_2, h_3, h_5\}, H_5(e_1, e_3) = \{h_1, h_2, h_3, h_5\}, H_5(e_1, e_4) = \{h_1, h_2, h_3, h_4\}, \\ H_5(e_1, e_5) &= \{h_2, h_3\}, H_5(e_2, e_2) = \{h_1, h_2, h_5\}, H_5(e_2, e_3) = \{h_1, h_2, h_3, h_5\} \end{aligned}$$

and

$$\begin{aligned} I_5(\neg e_1, \neg e_2) &= \{h_4\}, I_5(\neg e_1, \neg e_3) = \{h_4\}, I_5(\neg e_1, \neg e_4) = \emptyset, I_5(\neg e_1, \neg e_5) = \{h_4\}, \\ I_5(\neg e_2, \neg e_2) &= \{h_4\}, I_5(\neg e_2, \neg e_3) = \{h_4\} \quad \text{and so on.} \end{aligned}$$

Let $(F, G, A) \wedge (F_1, G_1, B) = (H_6, I_6, A \times B)$. Then

$$\begin{aligned} H_6(e_1, e_2) &= \{h_2\}, H_6(e_1, e_3) = \{h_3\}, H_6(e_1, e_4) = \{h_3\}, H_6(e_1, e_5) = \{h_2, h_3\}, \\ H_6(e_2, e_2) &= \{h_2, h_5\}, H_6(e_2, e_3) = \{h_1, h_5\} \end{aligned}$$

and

$$\begin{aligned} I_6(\neg e_1, \neg e_2) &= \{h_4, h_5\}, I_6(\neg e_1, \neg e_3) = \{h_2, h_4, h_5\}, I_6(\neg e_1, \neg e_4) = \{h_2, h_4, h_5\}, \\ I_6(\neg e_1, \neg e_5) &= \{h_1, h_4, h_5\}, I_6(\neg e_2, \neg e_2) = \{h_3, h_4\}, I_6(\neg e_2, \neg e_3) = \{h_2, h_3, h_4\} \end{aligned}$$

and so on.

A bipolar soft set may be represented by a pair of tables for each of the functions F and G respectively in a similar way as the tabular representation of soft sets is used by Maji et al. in [2]. We can also represent a bipolar soft set with the help of a single table by putting

$$a_{ij} = \begin{cases} 1 & \text{if } h_i \in F(e_j) \\ 0 & \text{if } h_i \in U - \{F(e_j) \cup G(\neg e_j)\} \\ -1 & \text{if } h_i \in G(\neg e_j) \end{cases}$$

where a_{ij} is the i th entry of j th column of the table whose rows and columns are labeled by houses and parameters respectively. The tabular representations of the bipolar soft set (F, G, A) are given by Table 1 and Table 2.

F	e₁	e₂	e₃	e₆
h_1	0	1	1	0
h_2	1	1	0	1
h_3	1	0	1	1
h_4	0	0	0	0
h_5	0	1	0	1

G	$\neg e_1$	$\neg e_2$	$\neg e_3$	$\neg e_6$
h_1	0	0	0	0
h_2	0	0	1	0
h_3	0	1	0	0
h_4	1	1	1	1
h_5	1	0	0	0

TABLE 1. Tabular Representaion of (F, G, A) using a Pair of Tables

(F,G,A)	e₁	e₂	e₃	e₆
h_1	0	1	1	0
h_2	1	1	-1	1
h_3	1	-1	1	1
h_4	-1	-1	-1	-1
h_5	-1	1	0	1

TABLE 2. Tabular Representaion of (F, G, A) using only one Table

Example 2. *Bipolar disorder is a serious psychological illness that can lead to dangerous behavior, problematic careers and relationships, and suicidal tendencies, especially if not treated early. A bipolar mood chart is a simple and yet effective means of tracking and representing patient's condition every month. Bipolar mood charts help patients, their families and their doctors to see probable patterns that might have been very difficult to determine. Bipolar children and their families will greatly benefit from mood charting and can expect early detection of symptoms and determination of proper treatments by their doctors. We construct a mood chart based upon a bipolar soft set as follows:*

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ be the set of days in which the record has been maintained i.e. $i = i$ th day under observation, for $1 \leq i \leq 7$. Let

$E = \{e_1, e_2, e_3, e_4, e_5\} = \{\text{Severe Mania, Severe Depression, Anxiety, Medication, Side effects}\}$ and

$\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4, \neg e_5\} = \{\text{Mild Mania, Mild Depression, No Anxiety, No Medication, No Side effects}\}$. Here the gray area is obviously the moderate form of parameters. Let the bipolar soft sets (F, G, E) describes the “daily record of the behavior” of Mr. X. Suppose that

$$\begin{aligned} F(e_1) &= \{1, 5\}, F(e_2) = \{1, 2, 3, 4, 7\}, F(e_3) = \{2, 4, 5, 6\}, \\ F(e_4) &= \{1, 2, 4, 5, 6, 7\}, F(e_5) = \{2, 3, 5, 7\}. \end{aligned}$$

and

$$\begin{aligned} G(\neg e_1) &= \{2, 6, 7\}, G(\neg e_2) = \{6\}, G(\neg e_3) = \{1, 7\}, G(\neg e_4) = \{3\}, \\ G(\neg e_5) &= \{1, 4, 6\}. \end{aligned}$$

The corresponding mood chart is given by the Table 3

(F, G, E)	e_1	e_2	e_3	e_4	e_5
1	1	1	-1	1	-1
2	-1	1	1	1	1
3	0	1	0	-1	1
4	0	1	1	1	-1
5	1	0	1	1	1
6	-1	-1	1	1	-1
7	-1	1	-1	1	1

TABLE 3. Tabular Representation of (F, G, E)

Lemma 1. Let (F, G, A) , (F_1, G_1, B) and (F_2, G_2, C) be any bipolar soft sets over a common universe U . Then the following are true

- (1) $(F, G, A)\alpha((F_1, G_1, B)\alpha(F_2, G_2, C)) = ((F, G, A)\alpha(F_1, G_1, B))\alpha(F_2, G_2, C)$,
- (2) $(F, G, A)\alpha(F_1, G_1, B) = (F, G, A)\alpha(F_1, G_1, B)$,

for all $\alpha \in \{\tilde{\cap}, \cap_{\mathcal{R}}, \tilde{\cup}, \cup_{\mathcal{R}}\}$.

Proof. Straightforward. \square

Lemma 2. Let (F, G, A) and (F_1, G_1, B) be two bipolar soft sets over a common universe U . Then the following are true

- (1) $(F, G, A)\tilde{\cup}(F_1, G_1, B)$ is the smallest bipolar soft set over U which contains both (F, G, A) and (F_1, G_1, B) .
- (2) $(F, G, A)\cap_{\mathcal{R}}(F_1, G_1, B)$ is the largest bipolar soft set over U which is contained in both (F, G, A) and (F_1, G_1, B) .

Proof. Straightforward. \square

Proposition 5. Let (F, G, A) , (F_1, G_1, B) and (F_2, G_2, C) be any bipolar soft sets over a common universe U . Then

- (1) $(F, G, A)\cap_{\mathcal{R}}((F_1, G_1, B)\tilde{\cup}(F_2, G_2, C)) = ((F, G, A)\cap_{\mathcal{R}}(F_1, G_1, B))\tilde{\cup}((F, G, A)\cap_{\mathcal{R}}(F_2, G_2, C))$,
- (2) $(F, G, A)\cap_{\mathcal{R}}((F_1, G_1, B)\cap_{\mathcal{R}}(F_2, G_2, C)) = ((F, G, A)\cap_{\mathcal{R}}(F_1, G_1, B))\cap_{\mathcal{R}}((F, G, A)\cap_{\mathcal{R}}(F_2, G_2, C))$,
- (3) $(F, G, A)\cap_{\mathcal{R}}((F_1, G_1, B)\cup_{\mathcal{R}}(F_2, G_2, C)) = ((F, G, A)\cap_{\mathcal{R}}(F_1, G_1, B))\cup_{\mathcal{R}}((F, G, A)\cap_{\mathcal{R}}(F_2, G_2, C))$,
- (4) $(F, G, A)\cup_{\mathcal{R}}((F_1, G_1, B)\tilde{\cup}(F_2, G_2, C)) = ((F, G, A)\cup_{\mathcal{R}}(F_1, G_1, B))\tilde{\cup}((F, G, A)\cup_{\mathcal{R}}(F_2, G_2, C))$,
- (5) $(F, G, A)\cup_{\mathcal{R}}((F_1, G_1, B)\cap_{\mathcal{R}}(F_2, G_2, C)) = ((F, G, A)\cup_{\mathcal{R}}(F_1, G_1, B))\cap_{\mathcal{R}}((F, G, A)\cup_{\mathcal{R}}(F_2, G_2, C))$,
- (6) $(F, G, A)\cup_{\mathcal{R}}((F_1, G_1, B)\tilde{\cap}(F_2, G_2, C)) = ((F, G, A)\cup_{\mathcal{R}}(F_1, G_1, B))\tilde{\cap}((F, G, A)\cup_{\mathcal{R}}(F_2, G_2, C))$,
- (7) $(F, G, A)\tilde{\cup}((F_1, G_1, B)\tilde{\cap}(F_2, G_2, C))\tilde{\supset}((F, G, A)\tilde{\cup}(F_1, G_1, B))\tilde{\cap}((F, G, A)\tilde{\cup}(F_2, G_2, C))$,
- (8) $(F, G, A)\tilde{\cup}((F_1, G_1, B)\cup_{\mathcal{R}}(F_2, G_2, C))\tilde{\supset}((F, G, A)\tilde{\cup}(F_1, G_1, B))\cup_{\mathcal{R}}((F, G, A)\tilde{\cup}(F_2, G_2, C))$,
- (9) $(F, G, A)\tilde{\cup}((F_1, G_1, B)\cap_{\mathcal{R}}(F_2, G_2, C)) = ((F, G, A)\tilde{\cup}(F_1, G_1, B))\cap_{\mathcal{R}}((F, G, A)\tilde{\cup}(F_2, G_2, C))$,
- (10) $(F, G, A)\tilde{\cap}((F_1, G_1, B)\tilde{\cup}(F_2, G_2, C))\tilde{\supset}((F, G, A)\tilde{\cap}(F_1, G_1, B))\tilde{\cup}((F, G, A)\tilde{\cap}(F_2, G_2, C))$,

- (11) $(F, G, A) \tilde{\cap} ((F_1, G_1, B) \cup_{\mathcal{R}} (F_2, G_2, C)) = ((F, G, A) \tilde{\cap} (F_1, G_1, B)) \cup_{\mathcal{R}} ((F, G, A) \tilde{\cap} (F_2, G_2, C)),$
 (12) $(F, G, A) \tilde{\cap} ((F_1, G_1, B) \cap_{\mathcal{R}} (F_2, G_2, C)) \tilde{\supset} ((F, G, A) \tilde{\cap} (F_1, G_1, B)) \cap_{\mathcal{R}} ((F, G, A) \tilde{\cap} (F_2, G_2, C)).$

Proof.

- (1) For any $e \in A \cap (B \cup C)$, we have following three disjoint cases:

- (i) If $e \in A \cap (B \setminus C)$, then

$$\begin{aligned} (F \cap_{\mathcal{R}} (F_1 \tilde{\cup} F_2))(e) &= F(e) \wedge F_1(e) \\ (G \cap_{\mathcal{R}} (G_1 \tilde{\cup} G_2))(\neg e) &= G(\neg e) \vee G_1(\neg e) \end{aligned}$$

and

$$\begin{aligned} ((F \cap_{\mathcal{R}} F_1) \tilde{\cup} (F \cap_{\mathcal{R}} F_2))(e) &= (F \cap_{\mathcal{R}} F_1)(e) \vee \emptyset \\ &= F(e) \wedge F_1(e) \\ ((G \cap_{\mathcal{R}} G_1) \tilde{\cup} (G \cap_{\mathcal{R}} G_2))(\neg e) &= (G \cap_{\mathcal{R}} G_1)(\neg e) \wedge \mathcal{U} \\ &= G(\neg e) \vee G_1(\neg e). \end{aligned}$$

- (ii) If $e \in A \cap (C \setminus B)$, then

$$\begin{aligned} (F \cap_{\mathcal{R}} (F_1 \tilde{\cup} F_2))(e) &= F(e) \wedge F_2(e) \\ (G \cap_{\mathcal{R}} (G_1 \tilde{\cup} G_2))(\neg e) &= G(\neg e) \vee G_2(\neg e) \end{aligned}$$

and

$$\begin{aligned} ((F \cap_{\mathcal{R}} F_1) \tilde{\cup} (F \cap_{\mathcal{R}} F_2))(e) &= \emptyset \vee (F \cap_{\mathcal{R}} F_2)(e) \\ &= F(e) \wedge F_2(e) \\ ((G \cap_{\mathcal{R}} G_1) \tilde{\cup} (G \cap_{\mathcal{R}} G_2))(\neg e) &= \mathcal{U} \wedge (G \cap_{\mathcal{R}} G_2)(\neg e) \\ &= G(\neg e) \vee G_2(\neg e). \end{aligned}$$

- (iii) If $e \in A \cap (B \cap C)$, then

$$\begin{aligned} (F \cap_{\mathcal{R}} (F_1 \tilde{\cup} F_2))(e) &= F(e) \wedge (F_1(e) \vee F_2(e)) \\ (G \cap_{\mathcal{R}} (G_1 \tilde{\cup} G_2))(\neg e) &= G(\neg e) \vee (G_1(\neg e) \wedge G_2(\neg e)) \end{aligned}$$

and

$$\begin{aligned} ((F \cap_{\mathcal{R}} F_1) \tilde{\cup} (F \cap_{\mathcal{R}} F_2))(e) &= (F \cap_{\mathcal{R}} F_1)(e) \vee (F \cap_{\mathcal{R}} F_2)(e) \\ &= (F(e) \wedge F_1(e)) \vee (F(e) \wedge F_2(e)) \\ &= F(e) \wedge (F_1(e) \vee F_2(e)) \\ ((G \cap_{\mathcal{R}} G_1) \tilde{\cup} (G \cap_{\mathcal{R}} G_2))(\neg e) &= (G \cap_{\mathcal{R}} G_1)(\neg e) \wedge (G \cap_{\mathcal{R}} G_2)(\neg e) \\ &= (G(\neg e) \vee G_1(\neg e)) \wedge (G(\neg e) \vee G_2(\neg e)) \\ &= G(\neg e) \vee (G_1(\neg e) \wedge G_2(\neg e)). \end{aligned}$$

Thus

$$(F, G, A) \cap_{\mathcal{R}} ((F_1, G_1, B) \tilde{\cup} (F_2, G_2, C)) = ((F, G, A) \cap_{\mathcal{R}} (F_1, G_1, B)) \tilde{\cup} ((F, G, A) \cap_{\mathcal{R}} (F_2, G_2, C)).$$

Similarly, we can check for the remaining parts. \square

Now we consider the collection of all bipolar soft sets over U and denote it by $\mathcal{BSS}(U)^E$ and let us denote its sub collection of all bipolar soft sets over U with

fixed set of parameters A by $\mathcal{BSS}(U)_A$. We note that this collection is partially ordered by inclusion. We conclude from above results that:

Proposition 6. $(\mathcal{BSS}(U)^E, \tilde{\cap}, \cup_{\mathcal{R}})$ and $(\mathcal{BSS}(U)^E, \tilde{\cup}, \cap_{\mathcal{R}})$ are distributive lattices and $(\mathcal{BSS}(U)^E, \cup_{\mathcal{R}}, \tilde{\cap})$ and $(\mathcal{BSS}(U)^E, \cap_{\mathcal{R}}, \tilde{\cup})$ are their duals respectively.

Proof. Follows from above results. \square

Proposition 7. $(\mathcal{BSS}(U)^E, \cap_{\mathcal{R}}, \tilde{\cup})$ is a bounded distributive lattice, with least element $(\Phi, \mathfrak{U}, \emptyset)$ and greatest element (\mathfrak{U}, Φ, E) , while $(\mathcal{BSS}(U)^E, \tilde{\cup}, \cap_{\mathcal{R}}, (\mathfrak{U}, \Phi, E), (\Phi, \mathfrak{U}, \emptyset))$ is its dual.

Proof. Follows from above results. \square

Proposition 8. $(\mathcal{BSS}(U)_A, \cap_{\mathcal{R}}, \tilde{\cup}) = (\mathcal{BSS}(U)_A, \tilde{\cap}, \cup_{\mathcal{R}})$ is a bounded distributive lattice, with least element (Φ, \mathfrak{U}, A) and greatest element (\mathfrak{U}, Φ, A) .

Proof. Follows from above results. \square

Proposition 9. Let (F, G, A) and (F_1, G_1, A) be two bipolar soft sets over a common universe U . Then

- (1) $((F, G, A)^c)^c = (F, G, A)$,
- (2) $(F, G, A) \subseteq (F_1, G_1, A)$ implies $(F_1, G_1, A)^c \subseteq (F, G, A)^c$.

Proof. 1. is straightforward

2. If $(F, G, A) \subseteq (F_1, G_1, A)$ then

$$F(e) \subseteq F_1(e) \text{ and } G_1(\neg e) \subseteq G(\neg e) \text{ for all } e \in A$$

implies that

$$(G_1, F_1, A) \subseteq (G, F, A).$$

$$\text{Hence } (F_1, G_1, A)^c \subseteq (F, G, A)^c.$$

\square

Proposition 10. $(\mathcal{BSS}(U)_A, \cap_{\mathcal{R}}, \cup_{\mathcal{R}}^c, (\mathfrak{U}, \Phi, A), (\Phi, \mathfrak{U}, A))$ is a De Morgan algebra.

Proof. Straightforward. \square

4. APPLICATION OF BIPOLAR SOFT SETS IN A DECISION MAKING PROBLEM

Decision making is an important factor of all scientific professions where experts apply their knowledge in that area to make decisions wisely. We apply the concept of bipolar soft sets for modelling of a given problem and then we give an algorithm for the choice of optimal object based upon the available sets of information. Let U be the initial universe and E be a set of parameters.

For the data analysis of a bipolar soft set, we shall use the single table representation of (F, G, E) as discussed on Page 8. It is understood that both types of tabular presentations are equivalent and may be used interchangeably where required. We shall adapt the following terminology afterwards:

Definition 23. The decision value of an object $m_i \in U$ is d_i , given by

$$d_i = \sum_j a_{ij}$$

where a_{ij} is (i, j) - th entry in the table of the bipolar soft set. We adjoin the column of decision parameter d having values d_i , with the table of bipolar soft set (F, G, E) to obtain the decision table.

We define the concept of indiscernibility relations associated with a bipolar soft set.

Definition 24. If (F, G, E) is a bipolar soft set over U along with the set E of choice parameters, then:

- (1) $\emptyset \neq F(e) \subset U$, $\emptyset \neq G(\neg e) \subset U$, such that $F(e) \cup G(\neg e) \neq U$, partition U into three classes;
- (2) If any one of $F(e)$ and $G(\neg e)$ is empty and the other one is a proper subset of U , then we have two classes of elements in U ;
- (3) If any one of $F(e)$ and $G(\neg e)$ is equal to U , it provides the universal equivalence relation $U \times U$.

In either of above three cases, these classes correspond to an equivalence relation on U . Consequently, we see that for each parameter $e \in E$, we have an equivalence relation on U . If this equivalence relation is denoted by $\sigma(e)$ for all $e \in E$, then (σ, E) is a soft equivalence relation over U . We denote

$$IND(F, G, E) = \bigcap_{e \in E} \sigma(e).$$

Clearly $IND(F, G, E)$ is itself an equivalence relation on U . The classes of $IND(F, G, E)$ are basic categories of knowledge presented by a bipolar soft set over U . We may further consider, that, $IND(E) = IND(F, G, E)$ where E is the set of parameters. We shall say that a decision table of (F, G, E) is consistent if and only if $IND(E) \subseteq IND(D)$, where $IND(D)$ is the equivalence relation that classifies U into the categories having the same decision values.

Definition 25. Let T be a consistent decision table of bipolar soft set (F, G, E) and T_γ be a decision table obtained from T by eliminating some column of $\gamma \in C$. Then γ is dispensable in T if

- (1) T_γ is consistent that is $IND(C - \gamma) = IND(D_\gamma)$
- (2) $IND(D) = IND(D_\gamma)$

Otherwise γ is indispensable or core parameter. The set of all core parameters of C is denoted by $CORE(C)$.

Algorithm 1. The algorithm for the selection of the best choice is given as:

- (1) Input the bipolar soft set (F, G, E) .
- (2) Input the set of choice parameters $C \subseteq E$.
- (3) Input the decision parameter $d \in D$, $d_i = \sum_j a_{ij}$ as the last column in the table obtained by choice parameters.
- (4) Rearrange the Input by placing the objects having the same value for the parameter d adjacent to each other.
- (5) Distinguish the objects with different values of d by double line.
- (6) Identify core parameters as defined in Definition 25. Eliminate all the dispensable parameters one by one, resulting a table with minimum number of condition parameters having the same classification ability for d as the original table with d .

(7) Find k , for which $d_k = \max d_i$.

Then m_k is the optimal choice object. If k has more than one values, then any one of m_k 's can be chosen.

Example 3. Let $U = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$ be the set of candidates who have applied for a job position of Office Representative in Customer Care Centre of a company. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\} = \{\text{Hard Working, Optimism, Enthusiasm, Individualism, Imaginative, Flexibility, Decisiveness, Self-confidence, Politeness}\}$ and $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \neg e_4, \neg e_5, \neg e_6, \neg e_7, \neg e_8, \neg e_9\} = \{\text{Negligent, Pessimism, Half-hearted, Dependence, Unimaginative, Rigidity, Indecisiveness, Shyness, Harshness}\}$. Here the gray area is obviously the moderate form of parameters. Let the bipolar soft sets (F, G, E) describes the “Personality Analysis of Candidates” as:

$$\begin{aligned} F(e_1) &= \{m_1, m_4, m_5, m_8\}, F(e_2) = \{m_1, m_2, m_3, m_4, m_8\}, \\ F(e_3) &= \{m_2, m_4, m_6, m_7, m_8\}, F(e_4) = \{m_6, m_7\}, F(e_5) = \{m_1, m_7, m_8\}, \\ F(e_6) &= \{m_4, m_5, m_6, m_7\}, F(e_7) = \{m_1, m_2, m_5, m_6, m_8\}, F(e_8) = \{m_1, m_6, m_8\}, \\ F(e_9) &= \{m_2, m_3, m_4, m_6, m_7\}. \end{aligned}$$

and

$$\begin{aligned} G(\neg e_1) &= \{m_6, m_7\}, G(\neg e_2) = \{m_5, m_6\}, G(\neg e_3) = \{\}, G(\neg e_4) = \{m_1, m_3, m_8\}, \\ G(\neg e_5) &= \{m_2, m_3, m_4, m_5, m_6\}, G(\neg e_6) = \{m_8\}, G(\neg e_7) = \{m_3, m_4\}, \\ G(\neg e_8) &= \{m_5\}, G(\neg e_9) = \{m_1, m_5\}. \end{aligned}$$

(1) Input the bipolar soft set (F, G, E) given by Table 4.

(F, G, E)	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
m_1	1	1	0	-1	1	0	1	1	-1
m_2	0	1	1	0	-1	0	1	0	1
m_3	0	1	0	-1	-1	0	-1	0	1
m_4	1	1	1	0	-1	1	-1	0	1
m_5	1	-1	0	0	-1	1	1	-1	-1
m_6	-1	-1	1	1	-1	1	1	1	1
m_7	-1	0	1	1	1	1	0	0	1
m_8	1	1	1	-1	1	-1	1	1	0

TABLE 4. Table of (F, G, E)

(2) Let $C = \{e_1, e_3, e_4, e_5, e_7, e_8\}$.

(3) The decision table of bipolar soft set (F, G, C) is given by Table 5.

We note that

$$\begin{aligned} IND(C) &= \{(m_1, m_1), (m_2, m_2), (m_3, m_3), (m_4, m_4), (m_5, m_5), (m_6, m_6), (m_7, m_7), (m_8, m_8)\} \\ &\subset \{(m_1, m_1), (m_2, m_2), (m_3, m_3), (m_4, m_4), (m_5, m_5), (m_6, m_6), (m_7, m_7), (m_8, m_8), \\ &\quad (m_5, m_4), (m_4, m_5), (m_6, m_7), (m_6, m_7)\} \\ &= IND(D). \end{aligned}$$

Hence the decision table is consistent.

(4) We rearrange the table according to the same values for d to obtain Table 6.

(F, G, C)	e_1	e_3	e_4	e_5	e_7	e_8	d
m_1	1	0	-1	1	1	1	3
m_2	0	1	0	-1	1	0	1
m_3	0	0	-1	-1	-1	0	-3
m_4	1	1	0	-1	-1	0	0
m_5	1	0	0	-1	1	-1	0
m_6	-1	1	1	-1	1	1	2
m_7	-1	1	1	1	0	0	2
m_8	1	1	-1	1	1	1	4

TABLE 5. Decision Table for (F, G, C)

(F, G, C)	e_1	e_3	e_4	e_5	e_7	e_8	d
m_8	1	1	-1	1	1	1	4
m_1	1	0	-1	1	1	1	3
m_6	-1	1	1	-1	1	1	2
m_7	-1	1	1	1	0	0	2
m_2	0	1	0	-1	1	0	1
m_4	1	1	0	-1	-1	0	0
m_5	1	0	0	-1	1	-1	0
m_3	0	0	-1	-1	-1	0	-3

TABLE 6. Table obtained after rearrangement of Rows according to the values of d

(5) We identify that

$$CORE(C) = C.$$

(6) We conclude from the values of d that $d_8 = \max d_i = 4$ and hence $k = 8$.

Thus m_8 is the optimal choice object and so m_8 is the best candidate for the position. In case that m_8 can not join the position, m_1 will be selected, and if m_1 will also not be able to join then either m_6 or m_7 may be selected.

We define another type of weighted table of the reduct bipolar soft set (F, G, C) . The motivation is, that, some of the parameters are of less importance than the other ones so they must be graded with lesser priority. For this reason, we suggest that the column of that parameter will have entries

$$b_{ij} = \begin{cases} a_{ij} \times w_j & \text{if } a_{ij} = 1 \\ 0 & \text{if } a_{ij} = 0 \\ a_{ij} \times (1 - w_j) & \text{if } a_{ij} = -1 \end{cases},$$

instead of 0 and 1 and -1 only, where a_{ij} are the entries in the table of the reduct bipolar soft set (F, G, C) .

Definition 26. The weighted decision value of an object $m_i \in U$ is

$$d_i = \sum_j b_{ij}.$$

The revised algorithm will be given now:

Algorithm 2. *The algorithm for the selection of the best choice is given as:*

- (1) *Input the bipolar soft set (F, G, E) .*
- (2) *Input the set of choice parameters $C \subseteq E$.*
- (3) *Find weighted table of the bipolar soft set (F, G, C) according to the weights decided.*
- (4) *Input the decision parameter $d \in D$, $d_i = \sum_j b_{ij}$ as the last column in the weighted table T_w .*
- (5) *Rearrange the input by placing the objects having the same value for the parameter d adjacent to each other.*
- (6) *Distinguish the objects with different values of d by double line.*
- (7) *Identify core parameters. Eliminate all the dispensable parameters one by one, resulting a table with minimum number of condition parameters having the same classification ability for d as the original table with d .*
- (8) *Find k , for which $d_k = \max d_i$.*

Then m_k is the optimal choice object. If k has more than one values, then any one of m_k 's can be chosen.

Now we solve the original problem using this revised algorithm. Suppose that the selection board sets the following weights for parameters of C and take start from the 3rd step as:

$$\begin{aligned}
 e_1 & : w_1 = 0.9 \\
 e_3 & : w_3 = 0.8 \\
 e_4 & : w_4 = 0.5 \\
 e_5 & : w_5 = 0.6 \\
 e_7 & : w_7 = 0.9 \\
 e_8 & : w_8 = 0.9
 \end{aligned}$$

The weighted decision table of bipolar soft set (F, G, C) is given by Table 7.

$(F, G, C)_w$	e_1	e_3	e_4	e_5	e_7	e_8	d
m_1	0.9	0	-0.5	0.6	0.9	0.9	2.8
m_2	0	0.8	0	-0.4	0.9	0	1.3
m_3	0	0	-0.5	-0.4	-0.1	0	-1
m_4	0.9	0.8	0	-0.4	-0.1	0	1.2
m_5	0.9	0.8	0	-0.4	0.9	-0.1	2.1
m_6	-0.1	0.8	0.5	-0.4	0.9	0.9	2.6
m_7	-0.1	0.8	0.5	0.6	0	0	1.8
m_8	0.9	0.8	-0.5	0.6	0.9	0.9	3.6

TABLE 7. Weighted Decision Table for (F, G, C)

We note that

$$\begin{aligned}
 IND(C) &= \{(m_1, m_1), (m_2, m_2), (m_3, m_3), (m_4, m_4), (m_5, m_5), (m_6, m_6), (m_7, m_7), (m_8, m_8)\} \\
 &= IND(D).
 \end{aligned}$$

Hence the decision table is consistent. We rearrange the table according to the descending values for d to obtain Table 8.

$(F, G, C)_w$	e_1	e_3	e_4	e_5	e_7	e_8	d
m_8	0.9	0.8	-0.5	0.6	0.9	0.9	3.6
m_1	0.9	0	-0.5	0.6	0.9	0.9	2.8
m_6	-0.1	0.8	0.5	-0.4	0.9	0.9	2.6
m_5	0.9	0.8	0	-0.4	0.9	-0.1	2.1
m_7	-0.1	0.8	0.5	0.6	0	0	1.8
m_2	0	0.8	0	-0.4	0.9	0	1.3
m_4	0.9	0.8	0	-0.4	-0.1	0	1.2
m_3	0	0	-0.5	-0.4	-0.1	0	-1

TABLE 8. Table of weighted Bipolar soft set (F,G,C) after Re-arrangement

We identify that

$$CORE(C) = C.$$

The values of d show that $d_8 = \max d_i = 4$ and hence $k = 8$.

Once again, m_8 is the optimal choice object and so m_8 is the best candidate for the position. We note that the difference occurs in the position of m_5 . In the first case m_5 was at 7th position out of 8 but under the weighted criteria m_5 takes 4th position over all.

Conclusion 1. *We have defined bipolar soft sets and various operations of union and intersection for them. We have also shown that the concept of bipolar soft sets is different and can not be subsumed by combining soft sets only. As we have discussed the concept of bipolarity for the set of parameters used for approximations of initial universe U , the idea may be extended for a further study of tri-polar and hence multipolar soft sets. Therefore, this paper gives an idea for the beginning of a new study for approximations of data with uncertainties.*

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